



## GATE

Subject : CS 2014\_Set-2 - SOLUTIONS

### TECHNICAL SECTION (Q. NO. 1 – 25) 1 MARKS

1. Required probability =  $\frac{4C_3 \cdot 6C_1}{10C_4} = 0.1143$

$\therefore P = 0.1143$

$100P = 11.43$

11.43 is the answer.

2.

Word (X)	No. of times it has occurred	P(X)	F(X) length of word
The	2	2/9	3
quick	1	1/9	5
brown	1	1/9	5
fox	1	1/9	3
jumps	1	1/9	5
over	1	1/9	4
lazy	1	1/9	4
dog	1	1/9	3
	Total words = 9		

$\therefore$  Expected length of the word drawn =  $\sum P(X).F(X)$

$$= \frac{2}{9} \times 3 + \frac{1}{9} \times 5 + \frac{1}{9} \times 5 + \frac{1}{9} \times 3 + \frac{1}{9} \times 5 + \frac{1}{9} \times 4 + \frac{1}{9} \times 4 + \frac{1}{9} \times 3$$

$$= 3.89$$

$\therefore$  3.89 is the answer.

3. In a bipartite graph  $K_{m, n}$

no. of vertices =  $m + n$

no. of edges =  $m \times n$

It is given that  $m + n = 12$

$$\therefore m = 12 - n$$

$$\begin{aligned} \therefore \text{no. of edges} &= m \times n \\ &= (12 - n)n \\ &= 12n - n^2 \end{aligned}$$

$$\therefore f(n) = 12n - n^2 \quad [\text{Maximize } f(n)]$$

$$f'(n) = 12 - 2n$$

$$f'(n) = 0 \text{ at } \boxed{n = 6}$$

$$f''(n) = -2 \quad \therefore \text{maxima}$$

$\therefore$  for  $n = 6$ ,

$$f(n) = 6 \times 6 = 36$$

$\therefore$  Maximum number of edges in a bipartite graph of 12 vertices is 36.

4.  $A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix}_{3 \times 1} \quad [1 \ 9 \ 5]_{1 \times 3}$

$$\therefore A = \begin{bmatrix} 2 & 18 & 10 \\ -4 & -36 & -20 \\ 7 & 63 & 35 \end{bmatrix}_{3 \times 3}$$

$$R_1 \rightarrow 2R_1 + R_2$$

$$\therefore A = \begin{bmatrix} 0 & 0 & 0 \\ -4 & -36 & -20 \\ 7 & 63 & 35 \end{bmatrix}$$

$$\therefore |A| = 0$$

0 is the answer.

5.  $f(x)$  has degree 3 and roots are

$x = 1, x = 2$  and  $x = 3$

$$\therefore f(x) = \alpha(x - 1)(x - 2)(x - 3)$$

$\alpha$  - constant

$$f(0) = \alpha(-1)(-2)(-3) = -6\alpha$$

$$f(4) = \alpha(3)(2)(1) = 6\alpha$$

$\therefore f(0) \cdot f(4) < 0$  as both have opposite signs

(A) is the correct option.

6. Self dual functions have the following properties:

(1) It is neutral (no. of min. terms = no. of max terms)

(2) It does not contain two mutually exclusive terms.

i.e. for  $n = 3$  (000, 111), (010, 101), (011, 100), (001, 110) are mutually exclusive terms.

$\therefore$  For a boolean function with 'n' variables, we have  $2^n$  min terms / max terms, out of which

$\frac{2^n}{2}$  are mutually exclusive i.e.  $2^{n-1}$ .

So we have  $2^{n-1}$  pairs and each pair has 2 choices.

$\therefore 2^{n-1}$  pairs have =  $2^{2^{n-1}}$  choices of self dual function

$\therefore 2^{2^{n-1}}$  self dual functions are possible.

(D) is the answer.

7.  $k = 2^n$

The given counter will be a k-bit ring counter as it counts exactly  $2^n$  states.

The output of the decoder will be 1 for only single output and rest all other outputs will be set to 0. So it will work as a ring counter counting k states.

$\therefore$  (C) is the answer.

8.  $(123)_5 = (x8)_y$

$x < y$

and  $y > 8$  .....(1)

$$5^2 \times 1 + 5 \times 2 + 5^0 \times 3 = xy + 8$$

---


$$38 = xy + 8$$

$$\therefore xy = 30$$

Possible values of x and y are

x	y
1	30
2	15
3	10
5	6
6	5
10	3
15	2
30	1

only these three sets of values satisfy the conditions in (1)

$\therefore$  3 solutions

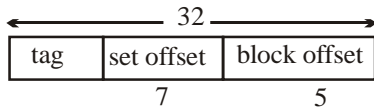
$\therefore$  3 is the answer.

9. Cache size = 16 kB  
 Block size = 8 words  
 1 word = 32 bits

$$\therefore \text{length of one block} = \frac{8 \times 32b}{8} = 32 \text{ bytes}$$

$$\text{Physical memory} = 4 \text{ GB} = 2^2 \cdot 2^{30} \text{B} = 2^{32} \text{B}$$

$\therefore$  For set associative memory, format is



$$\text{block offset} = \log_2(\text{Block size}) = \log_2(32) = 5$$

$$\# \text{cache lines} = \frac{\text{Cache size}}{\text{Block size}} = \frac{16 \text{ kB}}{32 \text{ B}} = 512 = 2^9$$

$$\# \text{sets} = \frac{\# \text{cache lines}}{\text{P-way}} = \frac{2^9}{4} = 128 = 2^7$$

$$\therefore \text{set offset} = \log_2(\# \text{sets}) = \log_2(2^7) = 7$$

$$\therefore \text{Tag bits} = 32 - (7 + 5) = 32 - 12 = 20 \text{ bits}$$

$\therefore$  20 is the answer.

10. Function (435)

The given function right shifts the given number until it is able to do so i.e. it divides the input by 2.

So the value returned is actually the no. of right shift (divide) operations that can be performed on this number.

In binary form,

$$(435)_{10} = (110110011)_2$$

$$\therefore \text{No. of successful right shift operations} = \text{no. of digits in binary form}$$

$$= 9$$

$\therefore$  9 is the return value.

11. As n is very large

$$\therefore {}^n C_3 = \frac{n(n-1)(n-2)}{6} \text{ will result in overflow and will return incorrect values.}$$

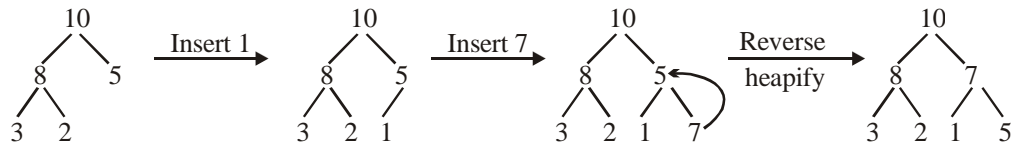
So we need to consider parts of the product to set it correctly.

As we know,  $n(n-1)$  will always be divisible by 2 so  $\frac{n(n-1)}{2}$  will result in correct answer followed

$$\text{by } \frac{(n-2)}{3}.$$

$\therefore$  (B) is the correct answer.

12. Heap is



∴ level order traversal of new heap

10 8 7 3 2 15

(A) is the answer.

13.  $T(n) = 2T\left(\frac{n}{2}\right) + \log n$

By using Master's theorem,

$a = 2, b = 2, k = 0, p = 1$

$b^k = 2^0 = 1$

∴  $a > b^k$

$T(n) = \theta(n^{\log_b a})$

$= \theta(n^{\log_2 2})$

∴  $T(n) = \theta(n)$

(A) is the answer.

14. BFS produces the shortest path from source to all other nodes in an unweighted and undirected graph.

(B) is the answer.

15.  $L_1 = \{a^n \mid n \geq 0\}$

$= \{\epsilon, a, aa, aaa, \dots\}$

∴  $L_1 = a^*$

$L_2 = \{b^n \mid n \geq 0\}$

$= \{\epsilon, b, bb, \dots\}$

∴  $L_2 = b^*$

Now,

$L_1.L_2 = a^*b^*$  which is a regular language.

∴ (I) is true and hence (II) is false (A) is the answer.

16. If  $A \leq_m B$ , then the following holds

(i) if B is decidable, then A is decidable

(ii) If A is undecidable, then B is undecidable.

(iii) If B is undecidable, then A is undecidable

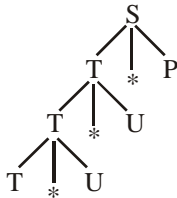
∴ The options (A), (B) and (C) are true.

However (D) is false. It is given that B is non REL.

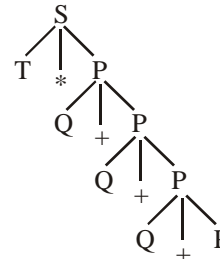
∴ It does not guarantee that A is also non REL.

∴ (D) is the answer.

17. From the given grammar,



∴ ‘\*’ is left associative



∴ ‘+’ is right associative.

**NOTE:** Also, the operator which is derived first in a parse tree has the least precedence

∴ Here ‘\*’ has least precedence

(B) is the correct option.

18. Dynamic memory allocation is done during run time.

The rest all are performed during compilation.

∴ (A) is the answer.

19. (C) is the correct answer.

20. #entries in FAT =  $\frac{\text{disk capacity}}{\text{Block capacity}}$

$$= \frac{100 \times 10^6 \text{ B}}{10^3 \text{ B}}$$

$$= 10^5$$

Total space consumed by overhead of each entry

$$= 4 \times 10^5 \text{ B}$$

∴ Maximum size of file that can be stored in FAT

$$= 100 \times 10^6 \text{ B} - 4 \times 10^5 \text{ B}$$

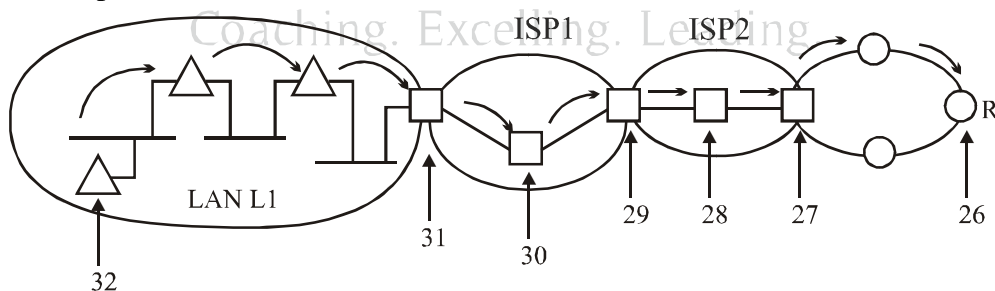
$$= (1000 - 4) \times 10^5 \text{ B}$$

$$= 996 \times 10^5 \text{ B}$$

$$= 99.6 \times 10^6 \text{ B}$$

∴ 99.6 is the answer.

21. Maximum no. of superkeys for a relation with  $n$  attributes =  $2^{n-1}$   
 Here  $n = 4$   
 $\therefore$  Maximum superkeys =  $2^{4-1} = 2^3 = 8$   
 Superkeys will be (with E as the key)  
 E, EF, EG, EH, EFG, EFH, EGH, EFGH  
 $\therefore$  8 is the answer.
22. For (Student Name, Student Age) to be a key, it should uniquely determine every tuple in a relation.  
 $\therefore$  X should not be 19 as there is another tuple with name as 'Shankar' and age '19'  
 $\therefore$  19 is the answer.
23. RIP and OSPF both are interior gateway protocols i.e., they both are used within an autonomous system.  
 RIP uses distance vector routing whereas OSPF uses link state routing.  
 $\therefore$  (A) is the answer.
24. Listen ( ) marks the socket referred to by sockfd as a passive socket i.e. a socket that will be used to accept incoming connection requests, using accept ( ).  
 A socket is created with socket ( ).  
 The socket is bound to a local address using bind ( ) .  
 And other sockets are connected to the bound address using the connect ( ) system call.  
 $\therefore$  (C) is the answer.
25. TTL value is decremented when the packet hops at each router. Routers work in network layer. Hence, we have to look for the number of times the packet enters the network layer. However, LANs work in Data link layer. So except LANs, the TTL value will be decremented at all other points.



- $\therefore$  26 is the answer.

## TECHNICAL SECTION (Q. NO. 26 – 55) 2 MARKS

26. For each link,

$$BW = 10^6 \text{ Bps}$$

$$\text{data size} = 10^3 \text{ B}$$

**Case I:**  $L = 10^3 \text{ B} + 100 \text{ B header} = 1100 \text{ B}$

$$\therefore T_t = \frac{L}{BW} = \frac{1100 \text{ B}}{10^6 \text{ Bps}} = 1.1 \times 10^{-3} \text{ sec} = 1.1 \text{ milli second}$$

$$\therefore \text{Total transmission time (at links } L_1, L_2 \text{ and } L_3) \\ = 3 \times 1.1 \text{ msec}$$

$$\therefore T_1 = 3.3 \text{ msec}$$

**Case II:**  $L = 100 \text{ B}$  and 10 packets + 100B header

$$\therefore \text{Total packet size} = 200 \text{ B}$$

$$\therefore T_t = \frac{200 \text{ B}}{10^6 \text{ Bps}} = 2 \times 10^{-4} \text{ sec} = 0.2 \text{ msec}$$

$$\therefore \text{Total transmission time for 10 packets (at } L_1, L_2 \text{ \& } L_3) \\ = 0.2 \times 3 + (10 - 1) \times 0.2 \text{ (using pipeline)} \\ = 0.6 + 1.8$$

$$\therefore T_2 = 2.4 \text{ msec}$$

**Case III:**

$$L = 50 \text{ B}$$
 and 20 packets + 100 B header

$$\therefore \text{Total packet size} = 150 \text{ B}$$

$$T_t = \frac{50 \text{ B}}{10^6 \text{ Bps}} = 1.5 \times 10^{-4} \text{ sec} = 0.15 \text{ msec}$$

$$\therefore \text{Total transmission time for 20 packets (at } L_1, L_2 \text{ \& } L_3) \\ = 0.15 \times 3 + (20 - 1) \times 0.15 \text{ (Using pipeline)} \\ = 0.45 + 2.85$$

$$\therefore T_3 = 3.3 \text{ msec}$$

$$\therefore T_1 = T_3, T_3 > T_2$$

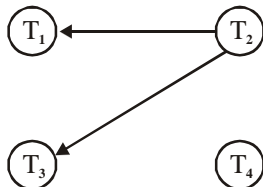
$\therefore$  (D) is the answer.

27. ●  $I_1$  cannot be learnt by the intruder as the URL and downloads take place at application layer whereas intruder sniffing at the router  $R_2$  cannot look up beyond network layer of OSI model.
- TCP port numbers are encapsulated in the payload field of IP data-gram. So it can be learnt by the intruder by sniffing at  $R_2$  alone.
- $I_3$  can be learnt by an intruder as IP addresses and routers work in network layer.
- $I_4$  cannot be learnt by the intruder because it is the function of data link layer of OSI model.
- $\therefore$  (C) is the answer.



28. A browser makes a separate request for each object of a webpage like images, java script, CSS. Even two distinct or similar images act as two objects and require two distinct requests. However, one TCP connection enough if these multiple resources are served from same server  
 $\therefore$  We need an additional request for the static embedded image.  
 So, (B) is the correct answer.

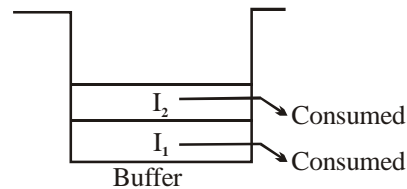
29. The precedence graph of S is



$\therefore$  It is conflict serializable as there is no cycle in precedence graph.  
 To check for recoverability, we check for W – R sequences.  
 On data item X, there is a WR sequence from T<sub>3</sub> to T<sub>4</sub>.  
 But T<sub>3</sub> is committed before commit of T<sub>4</sub>. On data item Y, there is a WR sequence from T<sub>2</sub> to T<sub>4</sub>. But T<sub>2</sub> is committed before commit of T<sub>4</sub>.  
 $\therefore$  The schedule S is recoverable  
 (C) is the answer.

30. Given that size (r(R)) < size (S(S))  
 if r(R) is in outer loop, then to perform join we will do  
 For each tuple in r(R), check for every tuple in S(S) if it satisfies join condition.  
 $\therefore$  We require a compulsory access of all the blocks in r(R) and for each r(R) we access all the tuples in S(S).  
 $\therefore$  We will require fewer block accesses if r(R) is in outer loop as |r(R)| < |S(S)|  
 (A) is the correct answer.

31. (A) is false  
 n = 0 – empty buffer



S = 1  
 Semwait (S); S = 0  
 $\therefore$  addToBuffer ( ) ;  $\therefore$  n = 1  
 Semsignal (S) ; S = 1  
 Semwait (S) ; S = 0  
 $\therefore$  addToBuffer ( ) ; n = 2 Semisignal (S) ;  $\therefore$  S = 1

Now try to consume the items  
 Semsignal (S); S = 0

semwait (n);  $\therefore n = 2 - 1 = 1$   $\therefore$  remove from buffer ( ) ;

Try to consume another item

Semsignal(s);  $\therefore S = 1$

semwait(S);  $S = 0$

Semwait(n);  $\therefore n = 1 - 1 = 0$   $\therefore$  remove from buffer( ) ;

$\therefore$  (B) is also false.

Now check (C)

Buffer is empty

$\therefore n = 0$

void consumer ( )

sem Wait (S);  $\therefore S = 0$

sem Wait(n); (blocked as n is already 0  $\therefore$  No successful wait operation)

Now we go to producer

void producer ( )

semWait(S) (blocked as consumer has already made S as 0)

$\therefore$  Both producer and consumer are in blocked state and hence deadlocked.

$\therefore$  (C) is the correct option.

32.

Process id	$t_c$	$t_{io}$	Arrival time
A	100 ms	500 ms	0
B	350 ms	500 ms	5
C	200 ms	500 ms	10

A, B and C each execute a loop of 100 iterations.

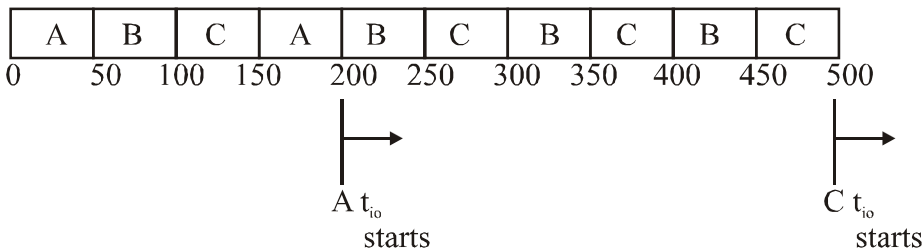
Using Round Robin and time quantum = 50 msec.

Ready Queue:

A B C A B C B C B

↓

Completes  $t_c$



$\therefore$  C will complete its I/O at  $t = 500 + 500 = 1000$  msec

1000 is the correct answer.

33. Optimal page replacement policy replaces that page which won't be used for the longest period of time.

The pages 1, 2,.....100 are accessed in that order and repeats the sequence thrice

∴ Sequence will be 1, 2, 3..... 100, 1, 2,..... 100, 1, 2, ..... 100, 1, 2, .....100.

∴ First 20 will cause page faults (1 – 20)

For page no. 21, 20 will be swapped as per optimal page replacement algorithm.

Similarly for page no. 22, 21 will be swapped. For 23, 22 will be swapped and so on.

Thus the optimal page replacement algorithm in this case works as most recently used.

∴ (D) is the correct answer.

34.  $t5 = x[i * 1024 + j * 32 + k * 4]$  .....(1)

Since k is multiplied by 4 (which is the size of an integer 32 bits = 4 bytes),

∴ X array must be an integer array.

∴ Options (C) and (D) are ruled out.

Let the array be X[L] [M][N]

X[i] [j] [k] th element in 3-D array is equivalent to X[i \* M \* N + j \* N + k] the element in one dimensional array.

∴ On comparing with equation (1), we get

$$j * 32 = j * N * 4 \quad (\text{Here 4 is size of int})$$

∴  $N = 8$

$$i * M * N * 4 = i * 1024$$

$$\therefore M = \frac{1024}{32} = 32$$

∴ we get  $N = 8$  and  $M = 32$

Only option (A) has  $N = 8$  and  $M = 32$

∴ (A) is correct

Coaching. Excelling. Leading.

35. M accepts a string of length 2014, which is finite.

∴ M halts on strings of length 2014.

However, it is not clear if M does not halt or constantly loops i.e. never halts or halts on a non final state.

∴ L is recursively enumerable and undecidable.

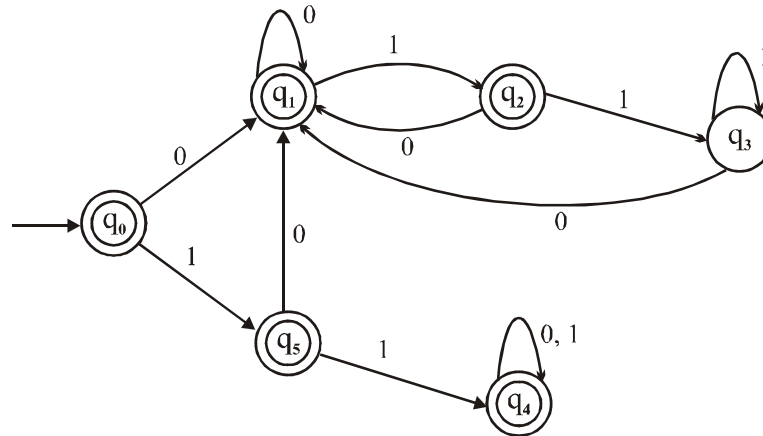
(B) is the answer.

36. (A) is true.

Though at first both L1 and L2 look non-regular, L1 is in fact regular.

$L1 = \{1, 0, 00, 11, 10, 0110, 01101, \dots\}$

DFA for L1 is



L2 is non-regular as DFA does not have memory to keep track of count of the occurrences of 000 and 111.

∴ (A) is correct.

37. A = qpqrr

B = pqprrp

3 such longest common subsequences are possible

LCS = qprr, qprr, qpqr

and length of LCS = 4

∴ x = 4 and y = 3

∴ x + 10y = 4 + 30 = 34

34 is the answer.

38. No. of comparisons for merging two sequences of length 'm' and 'n' in worst case is

$$(m + n - 1)$$

20, 24, 30, 35, 50

Merge 20 and 24  $(20 + 24 - 1) = 43$  comparisons

New sequence length = 44

∴ 30, 35, 44, 50

Merge 30 and 35  $(30 + 35 - 1) = 64$  comparisons

New sequence length = 65

44, 50, 65 Merge 44 and 50  $(44 + 50 - 1) = 93$  comparisons

New sequence length = 94

∴ 65, 94

Merge 65 and 94  $(65 + 94 - 1) = 158$  comparisons

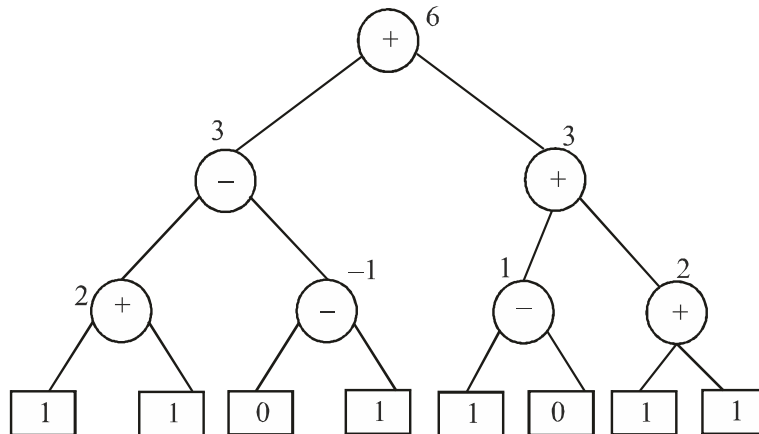
New sequence length = 159

∴ Total comparisons = 43 + 64 + 93 + 158

$$= 358$$

∴ 358 is the answer.

39. Maximum possible value will be when the values of leaves are as follows



∴ Maximum value is 6.

40. For  $f(q)$  to return  $q$

$$\text{abs}(x * x - 3) < 0.01$$

i.e. when  $x^2 - 3 = 0$

$$x^2 = 3$$

$$\therefore x = \sqrt{3} = 1.732$$

∴ 1.73 is the answer.

41. A queue can be implemented using this modified stack.

To dequeue an item, it takes 1 step i.e. simply  $\text{POP}()$  the item from the stack.

whereas to enqueue, it will take 3 steps

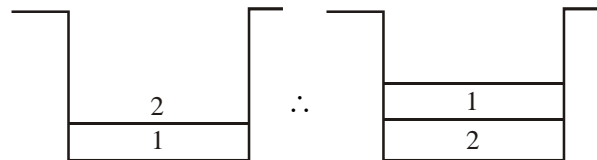
- (1) Reverse ( )
- (2) Push ( )
- (3) Reverse

Enqueue (1)

Coaching. Excelling. Leading.

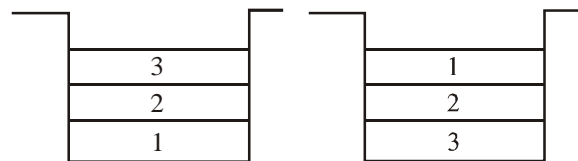
Enqueue (2)

(Push 2 & then reverse)



Enqueue (3)

Reverse



Push (3)

Reverse

as we need to put the elements in FIFO, whereas stack supports LIFO order.

(C) is the correct choice.

42. When  $j = 50$ , the condition inside if statement will become true.

$\therefore$  It will keep on calling  $f(i)$  infinitely

$\therefore$  (D) is the correct option.

43. Transferring blocks of data from main memory to cache memory is known as mapping.

(A) is false as if block size is small, TAG bits will be more and we would have less number of nearby addresses to refer in future. Hence, no better spatial locality.

(B) is false because lower block size implies more tag bits i.e. a larger cache tag which in turn increases complexity.

$\therefore$  More overhead by tag memory.

(C) is false because larger cache tag will not lower but increase cache hit time.

However option (D) is correct.

Smaller block size implies more memory blocks in cache at a particular time which in turn will reduce cache misses.

44. If associativity is doubled while keeping the capacity and block size constant, number of sets will be halved.

$\therefore$  width of set index decoder will decrease width of tag compositor will increase.

Also we need more ways of selection of multiplexer as associativity is doubled.

$\therefore$  (D) is the correct answer.

Also there is no relation between cache associativity and width of processor to main memory data bus.

45.  $X = -14.25$

$(-1)^s \times 1.F \times 2^{E-127}$  is the normalized form of representation of data in IEEE-754 32-bit floating point format

S – sign bit

F – fraction

E – exponent

As the given number is negative, sign bit will be 1

$\therefore S = 1$

14.25 in binary format is  $(1110.01)_2$

$\therefore$  Normalized form =  $1110.01 \times 2^0$

$$= \underbrace{1.11001}_F \times 2^3$$

F = 11001000..... 23 bits

Exponent,

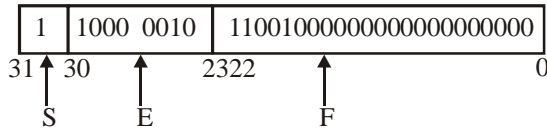
E – 127 = 3

$\therefore E = 130$

$\therefore E = 1000010$

Number  $N = (-1) \times 1.11001 \times 2^3$

In Hex format



$\therefore$  Grouping 4 bits from LSB to MSB, we get

1100	0001	0110	0100	0000	0000	0000	0000
C	1	6	4	0	0	0	0 H

$\therefore$  (A) is the answer.

46. In Newton's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 2.25x^2 - 4x - 2$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{(0.75 \times 2^3 - 2.2^2 - 2.2 + 4)}{2.25 \times 2^4 - 4.2 - 2}$$

$$\therefore x_1 = 2 - \left( \frac{-2}{-1} \right)$$

$$\therefore x_1 = 0$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0 - \left( \frac{4}{-2} \right)$$

$$x_2 = 2$$

$$\therefore x_2 = 2$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$\therefore x_3 = 0$$

$$x_4 = 2$$

$\therefore$  The method never converges to a solution, we alternately get  $x = 0$  and  $x = 2$

$\therefore$  (A) is the correct option.

$$47. \quad A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Characteristic equation of the matrix is

$$|A - \lambda I| = 0$$

$$\therefore |A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 & 0 & 1 \\ 0 & 1-\lambda & 1 & 1 & 0 \\ 0 & 1 & 1-\lambda & 1 & 0 \\ 0 & 1 & 1 & 1-\lambda & 0 \\ 1 & 0 & 0 & 0 & 1-\lambda \end{vmatrix}$$

Expanding the determinant by 1st row,

$$(1 - \lambda) \begin{vmatrix} 1-\lambda & 1 & 1 & 0 \\ 1 & 1-\lambda & 1 & 0 \\ 1 & 1 & 1-\lambda & 0 \\ 0 & 0 & 0 & 1-\lambda \end{vmatrix} + 1 \begin{vmatrix} 0 & 1-\lambda & 1 & 1 \\ 0 & 1 & 1-\lambda & 1 \\ 0 & 1 & 1 & 1-\lambda \\ 1 & 0 & 0 & 0 \end{vmatrix}$$

Now, expanding both determinants along last row,

$$-(1 - \lambda)(1 - \lambda) \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} + \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix}$$

$C \rightarrow C_1 + C_2 + C_3$ ;  $R_2 \rightarrow R_2 - R_1$ ;  $R_3 \rightarrow R_3 - R_1$  to both determinants

On solving, we get

$$-(1 - \lambda)^2 \lambda^2 (3 - \lambda) + \lambda^2(3 - \lambda) = 0$$

$$\lambda^2(3 - \lambda) [-(1 - \lambda)^2 + 1] = 0$$

$$\lambda^2(3 - \lambda) [-(1 - 2\lambda + \lambda^2) + 1] = 0$$

$$\lambda^2(3 - \lambda)(-1 + 2\lambda - \lambda^2 + 1) = 0$$

$$\therefore \lambda \cdot \lambda^2(3 - \lambda)(2 - \lambda) = 0$$

$$\lambda = 0, \lambda = 3 \text{ and } \lambda = 2$$

$$\therefore \text{Product of non-zero eigen values} = 3 \times 2 = 6$$

$$\therefore 6 \text{ is the answer.}$$



48. Out of 100 numbers,  
50 are divisible by 2,  
33 are divisible by 3  
and 20 are divisible by 5

$$\therefore n(2) = 50$$

$$n(3) = 33$$

$$n(5) = 20$$

$$n(2 \cap 3) = \frac{100}{2 \times 3} = 16.67$$

$$\therefore n(2 \cap 5) = \frac{100}{5 \times 2} = 10$$

$$\therefore n(2 \cap 5) = 10$$

$$n(3 \cap 5) = \frac{100}{3 \times 5} = 6.67$$

$$\therefore n(3 \cap 5) = 6$$

$$n(3 \cap 2 \cap 5) = \frac{100}{2 \times 3 \times 5} = \frac{100}{30} = 3.33$$

$$\therefore n(3 \cap 2 \cap 5) = 3$$

- $\therefore$  By principle of inclusion and exclusion,

$$\begin{aligned} n(2 \cup 3 \cup 5) &= n(2) + n(3) + n(5) - n(2 \cap 3) - n(3 \cap 5) - n(2 \cap 5) + (2 \cap 3 \cap 5) \\ &= 50 + 33 + 20 - 16 - 10 - 6 + 3 \\ &= 74 \end{aligned}$$

$$\therefore \text{No. of numbers not divisible by 2, 3 or 5} = 100 - 74 = 26$$

$$\therefore \text{Probability of such numbers} = \frac{26}{100} = 0.26$$

- $\therefore$  0.26 is the correct answer.

49.  $2014 = 2 \times 19 \times 53 = 2^1 \cdot 19^1 \cdot 53^1$

$$\therefore \text{No. of factors} = (1 + 1)(1 + 1)(1 + 1) = 8$$

- $\therefore$  8 is the answer.

50. S1 is true.

The subset of S that is larger than every other subset is the set S itself.

S2 is true.

The subset of S, that is smaller than every other set is ' $\phi$ '.

- $\therefore$  (A) is the answer.

51. For  $n = 3$



Cycle



Its complement

For  $n = 4$

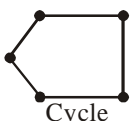


Cycle



Its complement

For  $n = 5$



Cycle



Its complement

They are isomorphic to each other as degree of each vertex is same in correspondences to each other.

Also the number of edges is same i.e. 5

And there is only one cycle of length 5 in both the case.

Try to imagine the complement in a plane

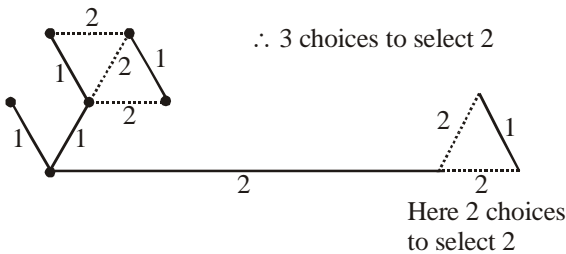
$\therefore n = 5$  is the answer.

52.



Spanning tree

Coaching. Excelling. Leading.



$\therefore$  3 choices to select 2

Here 2 choices to select 2

$\therefore$  We have  $3 \times 2$  choices to form a spanning tree.

So, no of different spanning trees = 6

6 is the answer.

53. (A)  $((a \rightarrow b) \wedge (b \rightarrow c)) \rightarrow (a \rightarrow c)$       use  $(a \rightarrow b \equiv \bar{a} + b)$

$$= ((\bar{a} + b) \cdot (\bar{b} + c)) \rightarrow (\bar{a} + c)$$

$$= (\bar{a}\bar{b} + \bar{a}c + bc) \rightarrow (\bar{a} + c)$$

$$= \overline{\bar{a}\bar{b} + \bar{a}c + bc} + \bar{a} + c$$

$$= \overline{\bar{a}\bar{b} \cdot \bar{a}c \cdot bc} + \bar{a} + c \quad (\text{By Demorgan's rule})$$

$$= (a + b\bar{c})(\bar{b} + \bar{c}) + \bar{a} + c$$

$$= \bar{a}b + \bar{a}\bar{c} + b\bar{c} + \bar{a} + c \quad (\bar{a} + a\bar{c} = \bar{a} + \bar{c})$$

$$= \bar{a}b + b\bar{c} + \bar{a} + \underline{\bar{c} + c}$$

$$= T + \bar{a} + b\bar{c} + \bar{a}b$$

$$= T(\text{Always true})$$

$\therefore$  It is a tautology

(B)  $(a \leftrightarrow bc) \rightarrow (\bar{b} \rightarrow (a \wedge c))$

$$= (\bar{a}\bar{b} + ab) \rightarrow (\bar{b} \rightarrow (ac))$$

$$= (\bar{a}\bar{b} + ab) \rightarrow (b + ac)$$

$$= \overline{(\bar{a}\bar{b} + ab)} + b + ac$$

$$= \overline{\bar{a}\bar{b} \cdot ab} + b + ac$$

$$= (a + b)(\bar{a} + \bar{b}) + b + ac$$

$$= \bar{a}b + \bar{a}\bar{b} + b + ac$$

$$= \bar{a}b + b(\bar{a} + 1) + ac$$

$$= \bar{a}b + b + ac$$

$$= b + a + ac$$

$$= a(1 + c) + b$$

$$= a + b$$

$\therefore$  It is not a tautology

$\therefore$  (B) is the answer      (No need to check other options).

Consider (C)

$$(a \wedge b \wedge c) \rightarrow (c \vee a)$$

$$= abc \rightarrow c + a$$

$$= \overline{abc} + c + a$$

$$\begin{aligned}
 &= \bar{a} + \bar{b} + \bar{c} + a + c \\
 &= T + \bar{b} \\
 &= T \quad \text{always true} \\
 \therefore &\text{ It is a tautology.}
 \end{aligned}$$

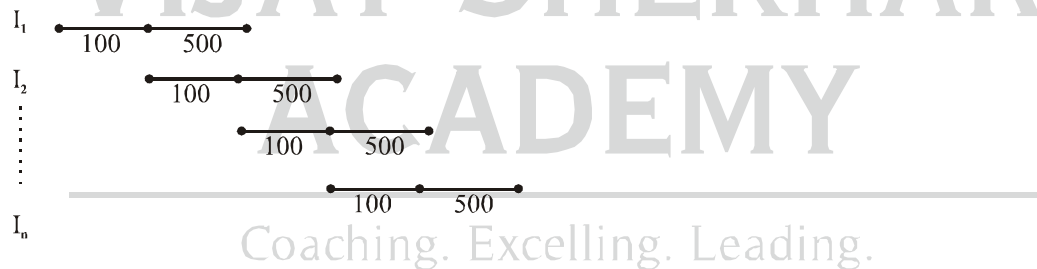
(D)  $a \rightarrow (b \rightarrow a)$

$$\begin{aligned}
 &= a \rightarrow (\bar{b} + a) \\
 &= \bar{a} + \bar{b} + a \\
 &= T + \bar{b} \\
 &= T \quad \therefore \text{ always true}
 \end{aligned}$$

So it is also a tautology.

54. The given query makes use of 'IN' operator. IN checks for all the tuples satisfying the IN condition. If there are multiple tuples of same attribute satisfying the IN condition, it will also return them. Thus the query actually performs the natural join of all in elements in R with distinct elements from S.  
 $\therefore$  (C) is the correct answer.

55. One write request takes 100 ns and 500 ns thereafter for storing the data. Thus, it forms a pipeline as it is given that different memory modules may overlap in time



$\therefore$  No. of stores initiated in 1 msec.

$$\begin{aligned}
 &= \frac{1 \text{ msec}}{100 \text{ n sec}} \\
 &= 10^4 \\
 &= 10000
 \end{aligned}$$

$\therefore$  10000 is the answer.

GENERAL APTITUDE

1. Colonialism was started by British in many countries including India.

∴ India is a post-colonial country now

(A) is the answer.

2. (B) Will be the correct choice.

did always comes with a present tense.

Remember that did + present tense verb = past tense verb.

3. radicate – remove completely, destroy utterly

distort – misrepresent, unclear representation

saturate – soak completely

utilize – use

∴ (A) is the answer.

4. Multiplies of 10 from 2 to 198 will be

10, 20,..... 100, 110, ..... 190

No. of terms = 19

$$\therefore \text{Average} = \frac{\text{Sum}}{19}$$

$$= \frac{19 [10+190]}{2 \cdot 19} \quad \text{Use sum of AP formula } S_n = \frac{n}{2} [a_0 + a_n]$$

$$= \frac{200}{2} = 100$$

∴ (B) is the answer.

Coaching. Excelling. Leading.

5. Let  $y = \sqrt{12 + \sqrt{12 + \sqrt{12}}}$

$$\therefore y = \sqrt{12 + y}$$

Squaring both sides,

$$y^2 = 12 + y$$

$$y - y - 12 = 0$$

$$(y - 4)(y + 3) = 0$$

$$y = 4 \text{ or } y = -3$$

∴ (C) is the answer.

6. (A) is not inferred from the passage as the first line mentions that the city had a German majority.  
 (B) is true because it is given that Kaliningrad is now a Russian territory despite being bordered by the Baltic Sea on the north and countries of Poland to the south and west and Lithuania to the east (i.e. it is not contiguous with the rest of Russia)  
 (C) and (D) are clearly not implied by the passage  
 $\therefore$  (B) is the answer.

7. The options (A), (B) and (C) contradict the given conclusion  
 (D) is the correct answer as both dengue and malarial fever is caused by mosquito bite.

8.  $|x^2 - 2x + 3| = 11$   
 i.e.  $x^2 - 2x + 3 = 11$  or  $x^2 - 2x + 3 = -11$   
 $\therefore x^2 - 2x - 8 = 0$   $x^2 - 2x + 14 = 0$   
 $(x - 4)(x + 2) = 0$   $a > 0$  and  $D = b^2 - 4ac < 0$   
 $\therefore x = 4$  or  $x = -2$   $\therefore$  Roots are imaginary  
 $y = |-x^3 + x^2 - x|$   
 For  $x = 4$ ,  $y = 52$   
 For  $x = -2$ ,  $y = 14$   
 $\therefore$  (D) is the answer.

9. In 2008,

$$\frac{M}{F} = \frac{2.5}{1} = \frac{5}{2}$$

Let there be 70 total students

$$\# \text{males} = \frac{5}{7} \times 70 = 50$$

$$\# \text{females} = \frac{2}{7} \times 70 = 20$$

It is given that females doubled in 2009.

$\therefore$  In 2009

$$\# \text{females} = 20 \times 2 = 40$$

$$\frac{M}{F} = \frac{3}{1} \quad \dots \text{Given}$$

$$\frac{M}{40} = \frac{3}{1}$$

$$\therefore M = 120$$

$$\therefore \% \text{ increase} = \frac{120 - 50}{50} \times 100$$

$$= \frac{70}{50} \times 100$$

$$= 140\%$$

∴ 140 is the answer.

10. At 6.22 am,  
we know that,

hour hand travels at  $\left(\frac{1}{2}\right)^\circ$  per minute

Minute hand travels at  $6^\circ$  per minute

∴ distance covered by hour hand in 22 minutes

$$= \frac{1}{2} \times 22 = 11^\circ$$

i.e.  $11^\circ$  after 6 O' clock

(as it will cover  $180^\circ$  at 6 : 00 hours)

∴ Total distance covered is  $191^\circ$

distance covered by minute hand in 22 minutes

$$= 6 \times 22 = 132^\circ$$

∴ Difference between their angles =  $191^\circ - 132^\circ$

=  $59^\circ$  (is closest to  $60^\circ$ )

∴ (A) is the answer.

VIJAY SHEKHAR  
ACADEMY

---

Coaching. Excelling. Leading.