GATE

Subject : CS 2014_Set-3 - SOLUTIONS

TECHNICAL SECTION (Q. NO. 1 – 25) 1 MARKS

1. Let a : Good mobile phones
   b: Cheap mobile phones
   $\therefore$ $P = a \rightarrow \neg b = \neg a \vee \neg b$
   $Q = b \rightarrow \neg a = \neg b \vee \neg a$
   $\therefore$ $P \equiv Q$
   Since, P and Q are equivalent to each other they will definitely imply each other.
   $\therefore$ (D) is the answer.

2. Option (A) is not true for all subsets A and B of X.
   It is true only if A and B are disjoint and mutually exhaustive subsets of X.
   Let X = {a, b, c} and Y = {1, 2}
   f(a) = 1, f(b) = 2, f(c) = 1
   Let A = {a, b} & B = {b, c}
   $f(A \cap B) = f(b) = 2$
   and $f(A) \cap f(B) = \{1, 2\} \cap \{2, 1\} = \{2, 1\}$
   $\therefore$ Option (B) is false.
   Now let A = {a, b} & B = {b, c}
   $|f(A \cap B)| = |\{1\}| = 1$
   $\min \{|f(A)|, |f(B)|\} = \min \{2, 2\} = 2$
   $\therefore$ (C) is false.
   (D) is the correct answer.
   for a function f: $X \rightarrow Y$, for every value of X, there is a unique Y.
   Let S = {1, 2} and T = {2}
   $f^{-1}(S \cap T) = b$; for, $f^{-1}(S) \cap f^{-1}(T) = \{a, c, b\} \cap \{b\} = \{b\}$
   $\therefore$ $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$ is true.
   for any subsets S and T of Y.

3. The order of the subgroup is such that it divides the order of group.
   ∴ For a group of order 15, subgroup can have order as 1, 5, 3 or 15.
   But it is given that order is atleast 4 and L! = G
   i.e. order of subgroup is not 15.
   ∴ The size of L is 5.
   5 is the answer.

4. Trace of a matrix is the sum of its diagonal elements.
   (A) is the correct option.

5. $D(V_1 + V_2) = D(V_1) + D(V_2) - D(V_1 \cap V_2)$
   $\therefore \quad 6 = 4 + 4 - D(V_1 \cap V_2)$
   $\therefore \quad D(V_1 \cap V_2) = 8 - 6 = 2$
   2 is the answer.

6. $\int_0^{2\pi} |x \sin x| dx = k\pi$ given

   Let $I = \int_0^{2\pi} |x \sin x| dx$

   $= \int_0^{\pi} x \sin x + \int_0^{2\pi} -x \sin x \, dx$

   $= \int_0^{\pi} x \sin dx - \int_0^{2\pi} x \sin x \, dx$

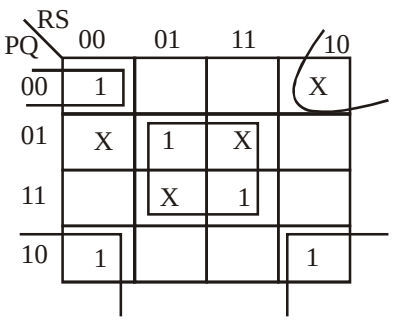
   $= [-x \cos x + \sin x]_0^{\pi} - [-x \cos x + \sin x]_{\pi}^{2\pi}$

   $= \pi - (-3\pi)$

   $= 4\pi$

   $\therefore \quad k = 4$

7. F(P, Q, R, S) = ∈(0, 2, 5, 7, 8, 10, 13, 15) 2, 7, 8 and 13 are don't care terms.



   $\therefore \quad F = \overline{Q}\overline{S} + QS$
   (B) is the correct answer.

8. The given function f is a multiplexer with 'x' as the select line.

   $\therefore \quad f = ax + bx'$

   (C) is the answer.

9. We know that,

$$\text{frequency} = \frac{1}{\text{cycle time}}$$

Cycle time of a processor is it maximum delay.

For P_1, $f = \frac{1}{2\,ns} = 0.5$ GHz

For P_2, $f = \frac{1}{1.5\,ns} = 0.67$ GHz

For P_3, $f = \frac{1}{1ns} = 1$ GHz

For P_4, $f = \frac{1}{1.1ns} = 0.909$ GHz

$\therefore$ P_3 has highest clock frequency

(C) is the answer.

10. The output is the matrix A itself. Since the loop runs for all values of i, j the elements in the matrix A are swapped twice, which retains the matrix A.

    (A) is the correct answer.

11. $$P(X) = X^5 + 4X^3 + 6X + 5$$
$$= X(X^4 + 4X^2 + 6) + 5$$
$$= X(X(X^3 + 4X) + 6) + 5$$
$$= X(X(X(X^2 + 4)) + 6) + 5$$

    Let t = X * X

    and we need to perform 3 additions and 4 multiplications (including the one in t)

    $\therefore$ Total 7 arithmetic operations are needed

    $\therefore$ 7 is the answer.

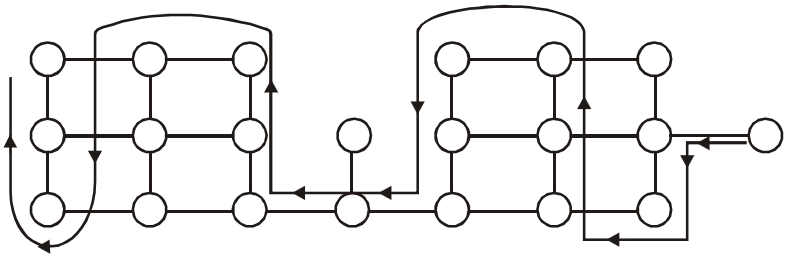
12. Inorder traversal works as follows:

    (i) Visit the left sub tree

    (ii) Visit the root

    (iii) Visit the right sub tree

    $\therefore$ (A) is the correct answer.

13. 

∴ Maximum depth = 19 (including initial call)

19 is the answer.

14. In the worst case, the chosen central element may be an extreme element.

i.e. Recurrence relation will become

$$T(n) = T(n-1) + \theta(n)$$

∴ $T.C = O(n^2)$

15. The string 'bab' of length 3 is not accepted by the given regular language.

∴ 3 is the answer.

16. A set is uncountable of its cardinal number is larger than that of the set of all natural numbers.

∴ (C) is the answer.

17. Intermediate code is a machine-independent code. It is used to improve the performance of code generation (i.e. optimizing time and space complexity) and to increase the chances of reusing the machine independent code optimizer in other compliers.

∴ (C) is the correct option.

18. (1) is true as to implement recursion, we need dynamic allocation of memory.

(2) is false.

(3) is true as number of recursive calls is not known prior to execution.

(4) is false as heap is used to dynamically allocate memory and is not needed for recursion.

∴ (D) is the answer.

19. Coupling is the degree of interdependence between software modules. Whereas cohesion is the degree to which the elements of a module belong together. For a good software design, it is always desirable to have high cohesion and low coupling.

∴ (B) is the answer.

20. Using LRU (Least recently used) page replacement policy,

#page frames = 3

4	7	6	1	7	6	1	2	7	2
4	4	4	1	1	1	1	1	1	1
	7	7	7	7	7	7	2	2	2
		6	6	6	6	6	6	7	7
				Hit	Hit	Hit			Hit

$\therefore$ #page faults = 6

6 is the answer.

21. The given expression selects the tuples which satisfy the conditions F1 and F2 and ultimately project the set of attributes in Al.

$\therefore$ The equivalent optimized expression will be

$\pi_{Al}(\sigma_{(F_1 \wedge F_2)}(r))$

(A) is the correct answer.

22. A prime attribute or a key attribute is an attribute contained in some candidate key of R.

$\therefore$ (B) is the answer.

23. (A), (C) and (D) are correctly matched.

Bit synchronization is done by Physical layer and not data link layer.

$\therefore$ (B) is the answer.

24. The delimiter flag is 01111110

$\therefore$ After every 5 1's, a 0 is stuffed.

Given output is 011111$\boxed{0}$0101

The highlighted 0 is stuffed.

So the input string must be 0111110101

$\therefore$ (B) is the answer.

25. (i) TTL is altered i.e. decremented at every hop by the router to avoid continuous or indefinite looping of packets.

(ii) Checksum of the packet is also altered as it is recalculated at every TTL.

(iii) Fragmentation offset is changed when a packet is fragmented i.e. when its size is greater than the MTU (Maximum Transmission Unit)

$\therefore$ (D) is the answer.

TECHNICAL SECTION (Q. NO. 26 – 55) 2 MARKS

26. Given IP address = 131.23.151.76

The first entry in the routing table is 131.16.0.0/12

∴ Net mask = 12 bits (i.e. left most 12 bits are 1s)

= 255.240.0.0

255.240.0.0
∧ 131.23.151.76
131.16.0.0

∴ Matching with the given address.

Now, consider for the second entry i.e. 131.28.0.0/14

∴ Net mask = 14 bits

= 255.252.0.0

255.252.0.0
∧ 131.23.151.76
131.20.0.0

∴ Not matching.

For third entry i.e. 131.19.0.0 / 16

∴ Net mask = 16 bits

255.255.0.0
∧ 131.23.151.76
131.23.0.0

∴ Not matching

For fourth entry, i.e. 131.22.0.0/15

Net mask = 15 bits

255.254.0.0
131.23.0.0
131.22.0.0

∴ matching with the given address.

Out of interface 1 and 3, we will select the longest matching prefix i.e. of interface 1.

∴ 1 is the answer.

So, out of the given entries in the routing table, the interfaces 1 and 3 are matching.

Hence, we need to consider the most subnetted mask i.e. longest matching prefix among the two. The interface 1 has the longest matching prefix.

∴ (A) is the answer.

27. Wrap around time is the time in which all the hosts generate all possible IDs.

Total IDs possible with 50 bit = 2^{50}.

For a IPv4 host, each host has a unique address and we know that IPv4 address is 32 bits.

$\therefore$ 2^{32} unique addresses

Also each host generates 1000 unique addresses per second.

$\therefore$ Total IDs per second $= 2^{32} \times 1000$

$= 2^{32} \times 2^{10}$

$= 2^{42}$

$$\text{Wrap around time} = \frac{\text{Total possible IDs}}{\text{Total generated IDs per second}}$$

$$= \frac{2^{50}}{2^{42}}$$

$= 2^8$

$= 256$ seconds

$\therefore$ 256 is the answer.

28. IP packet size = 4404 B

IP header = 20B

$\therefore$ Total packet size = 4404 + 20 = 4424B

MTU = 1500B (including header)

$\therefore$ data in MTU = 1500 – 20 = 1480 B

$\therefore$ No. of fragments $= \left\lceil \frac{4424}{1480} \right\rceil$

$= 3$

Since there are only three fragments, the MF (More fragment) bit for the third fragment will be 0.

$\therefore$ (B) and (C) are ruled out.

Fragment	**Data**	**Fragment offset**
1	0 - 1479	0/8 = 0
2	1480 - 2959	1480 /8 = 185
3	2960 - 4404	2960 / 8 = 370

$\therefore$ (A) is the answer.

29.

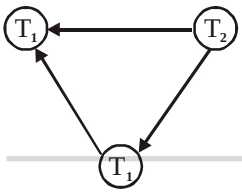
S_1

T_1	T_2	T_3
r(X)		
		r(Y)
	r(Y)	
	r(Z)	
		w(Y)
	w(Z)	
r(Z)		
w(X)		
w(Z)		

S_2

T_1	T_2	T_3
r(X)		
		r(Y)
	r(Y)	
		r(X)
r(Z)		
	r(Z)	
		w(Y)
w(X)		
	w(Z)	
w(Z)		

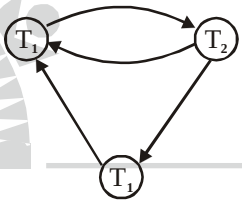
∴ Precedence graph for S_1 is



∴ No directed cycle

∴ Serializable

Precedence graph for S_2 is



There is a cycle

∴ Not serializable

(A) is the answer.

30. When we execute the subquery

Π_{empId}(employee $\bowtie_{(emp.\ ID = e.ID) \wedge (empAge \leq depAge)}$ dependent)

we get emp Ids of all employees whose age is less than or equal to the age of their dependents. When we subtract this result from Π_{empId}(employee) we get the empIds of employees whose age is greater than all of his / her dependents.

∴ (D) is the answer.

31.

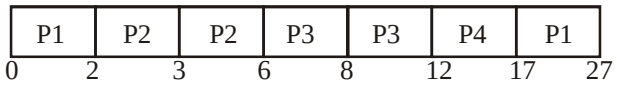
Program	Max need	∴ Peak demands
P1	3	3 – 1 = 2
P2	3	3 – 1 = 2
P3	3	3 – 1 = 2
		6 + 1 (extra tape unit for completion of atleast one process)

∴ We need atleast 7 tape units

∴ 7 is the answer.

32.

Process	A.T.	B.T.	C.T.	TAT	WT
P1	0	12	27	27	15
P2	2	4	6	4	0
P3	3	6	12	9	3
P4	8	5	17	9	4



$\therefore$ Avg. waiting time = $\frac{15+0+3+4}{4}$

$= \frac{22}{4}$

$= 5.5$ msec

$\therefore$ 5.5 is the answer.

33. EMAT = 0.6 (10 + 80) + 0.4 (10 + 2 × 80)

= 0.6(90) + 0.4 (170

= 122 msec

$\therefore$ 122 is the answer.

34. a = b + c

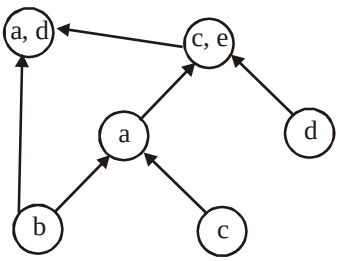
c = a + d $\quad\therefore$ c = b + c + d

d = b + c

e = d – b $\quad$ e = b + c – b = c $\quad\therefore$ e = c

a = e + b $\quad\therefore$ a = d – b + b = d $\quad\therefore$ a = d

$\therefore$ DAG is as follows



$\therefore$ #nodes = 6

#edges = 6

$\therefore$ (A) is the answer.

35. It is undecidale if a given context free grammar is ambiguous.

Rest all given statements are decidable.

$\therefore$ (A) is the answer.

36. $L_1 = \{0^n1^n \mid n \geq 0\}$

It is a DCFL

For every 0, push it onto the stack and for every 1, pop one 0.

$L_2 = \{w\ c\ w^r \mid w \in \{0, 1\}^*\}$

It is a DCFL

We push all the alphabets until we encounter C and pop each alphabet of w until w^r gets exhausted.

$L_3 = \{ww^r \mid w \in \{0, 1\}^*\}$

It is not a DCFL as it is difficult to determine the string w before hand. Also the PDA may have multiple transitions over the same input alphabets.

$\therefore$ Only L_1 and L_2 are DCFLS

$\therefore$ (C) is the answer.

37. From 9 on a number line, we can move forward in either of the two ways:

(i) Move right by a unit distance i.e. move to 10.

(ii) Take a shortcut from 9 to 15.

$\therefore$ $T(9) = \min (T(10), T(15))$

$\therefore$ $yz = 10 \times 15 = 150$

150 is the answer.

38. 2CNFSAT is solvable in polynomial time but 3CNFSAT is not because it is an NP complete problem.

2 CNFSAT can be reduced to strongly connected components problem, which has a polynomial time solution.

$\therefore$ (B) is the answer.

39. The solution is to traverse the balanced binary tree from root until we find a number that is in the given range i.e. between L and H.

For every node that is between L and H, we will check for its children nodes i.e. left node and right node if they lie between L and H and check recursively for further nodes.

Thus, the time taken will be O(m + log n) where 'n' is the total number of nodes in the tree and there are 'm' such nodes between L and H.

$\therefore$ $a = 0,\ b = 1,\ c = 1,\ d = 0$

$\therefore$ $a + 10b + 100c + 1000d = 110$

110 is the answer.

40. Given a hash table with 100 slots, the probability that the first 3 slots are unfilled after first 3 insertions

= P(first 3 slots empty after first insertion) *

= P(first 3 slots empty after second insertion) *

P(first 3 slots empty after third insertion)

$$= \frac{97}{100} \times \frac{97}{100} \times \frac{97}{100}$$

The probability of placing an item in a particular slot is independent for each slot.

$\therefore$ (A) is the answer.

41. The given function returns all the nodes in a tree whose leftmost as well as rightmost child is NULL. i.e. it returns the number of leaf nodes in a tree.

$\therefore$ (D) is the answer.

42. The given code is an implementation of binary search.

'k' calculates the mid of the list.

If the number to be searched (x here) is less than or equal to list A[k], then it traverses to left half of the list, else to the right half. And this goes on recursively until the list A[k] is scanned completely.

$\therefore$ (B) is the answer.

43. In old design,

t_p = max stage delay = 2.2 ns

80% instructions take one clock cycle and 20% incur stalls (i.e. they will take 3 clock cycles as branch is executed at the end of EX stage).

$\therefore$ Execution time P = (0.8 × 1 + 0.2 × 3) × 2.2

= 3.08 nsec

In new design, number of stages is 8

t_p = 1 nsec

80% instructions take one clock cycle and the rest 20% incur stalls (i.e. they will take 6 clock cycles as branch is executed at the end of EX2 stage)

$\therefore$ Execution time Q = (0.8 × 1 + 0.2 × 6) × 1

= 2 nsec

$\therefore$ P/Q = 3.08 / 2 = 1.54

1.54 is the answer.

44. Total instructions = 100 (fetch) + 60 (read) + 40 (write)

= 200

Fetching an operation is similar as reading from memory.

$\therefore$ Time taken for 100 fetch operations

$= 100 \times 0.9 \times 1 + 100 \times 0.1 \times 5$

$= 140$ nsec

Time taken for 60 read operations

$= 60 \times 0.9 \times 1 + 60 \times 0.1 \times 5$

$= 84$ nsec.

Time taken for 40 write operations

$= 40 \times 0.9 \times 1 + 40 \times 0.1 \times 10$

$= 112$ nsec.

$\therefore$ Total time taken $= 140 + 84 + 112$

$= 336$ nsec

$\therefore$ Average time taken for 200 instructions

$$= \frac{336}{200}$$

$= 1.68$ nsec

$\therefore$ 1.68 is the answer.

45. Truth table for JK FF is

J	K	Q_n
0	0	Q
0	1	0
1	0	1
1	1	$\overline{Q}$

For the given circuit, Q2 Q1 Q0 = 0 0 0 initially

Present State			Inputs						Next State		
Q2	Q1	Q0	J2	K2	J1	K1	J0	K0	Q2N	Q1N	Q0N
0	0	0	1	0	0	1	0	1	1	0	0
1	0	0	1	0	1	0	0	1	1	1	0
1	1	0	0	0	1	0	1	1	1	1	1

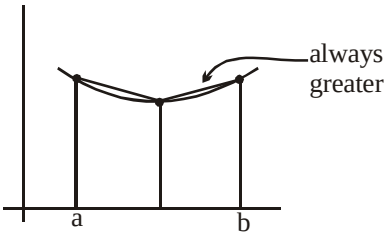
$\therefore$ (C) is the answer

46. Output by simpson's rule is very close to the true solution as compared to Trapezoidal rule

- Trapezoidal rule gives exact result while integrating polynomial upto degree 1.
- Simpson's $(3/8)^{th}$ rule gives exact result upto degree 3 whereas simpson's $(1/3)^{rd}$ rule gives exact result upto degree 2.

$\therefore$ Statement II is correct.

For statement I,



$\therefore$ I is also true.

$\therefore$ (C) is the answer.

47. $\int_0^{\pi} x^2 \cos x \, dx$

Use Leibnitz Rule,

$= \left[x^2 \sin x - (2x)(-\cos x) + (2)(-\sin x) \right]_0^{\pi}$

$= (-2\pi) - 0$

$= -2\pi$

$\therefore$ (A) is the answer.

48. A and B are mutually exclusive events.

$\therefore$ P(A) + P(B) = 1 .....(1)

Maximum value of P(A).P(B)

$=$ P(A).(1 – P(A))

Let P(A) = x

$\therefore$ $f(x) = x(1 - x) = x - x^2$

$\therefore$ $f'(x) = 1 - 2x$

$f'(x) = 0$ at $x = 0.5$

$f''(x) = -2 < 0$

$\therefore$ Maxima at $x = 0.5$

$\therefore$ $f(0.5) = 0.5(1 - 0.5) = 0.25$

0.25 is the answer.

49. The given function is $f(f(i)) = i; 0 \le i \le 2014$

$f(i) = i$ is an identify function.

Since the domain and co-domain are equal, there be for every such function such that

$f(i) = i$ for some i

$\therefore$ Q is true.

It is not necessarily possible $f(i) = 1$ for every such function

$\therefore$ P is false.

The function f(f(i)) = i is onto as i will take 2015 distinct values (0 to 2014) and domain and co-domain of the given function is same.

Hence, range = codomain

$\therefore$ The function is an onto function.

$\therefore$ R is true.

(B) is the answer

50. $x * x = e$ i.e. x is its own inverse.

$y * y = e$ similarly y is its own inverse.

$(x * y) * (x * y) = e$ i.e. x * y is its own inverse

$(y * x) * (y * x) = e$ i.e. y * x is its own inverse

we can also write

$x * y * e = e * e$ as

$(x * y) * (y * x) = e$

i.e. x * y and y * x are inverse of each other.

Also, they are inverse of their own.

We know that for a group, each element has a unique inverse.

This implies that (x * y) and (y * x) is one element

i.e. $x * y = y * x$

$\therefore$ The elements in a group are {e, x, y, x * y}

4 is the answer.

51. For 'n' vertices and 'k' connected components, minimum edges in a graph = n – k

maximum edges in a graph = $\frac{(n-k)(n-k+1)}{2}$

$\therefore$ (C) is the answer.

52. As per Euler's formula for planar graphs:

$f = e - v + 2$ .....(1)

f – no. of faces / regions

e – no. of edges

v – no. of vertices

Given that degree of vertices is atleast 3,

$\therefore$ $3v \geq 2e$.....(2) [Sum of degree of vertices is twice the edges]

From (1) and (2)

$$f \geq \frac{3v}{2} - v + 2$$

$$f \geq \frac{v}{2} + 2$$

$\therefore$ (A) is the answer

53. "Not all rainy days are cold"

The sentence simply means that there exists a rainy day that is not cold i.e.

$$\exists d(\text{Rainy}(d) \wedge \sim \text{cold}(d))$$

$\therefore$ (D) is the answer.

54. 'NOT EXISTS' returns the tuples that do not satisfy the given condition.

i.e. it returns the tuples that are false for the given condition.

$\therefore$ It returns the names of all the employees with all their customers having a 'GOOD' rating.

$\therefore$ (D) is the answer.

55. $F(P, Q) = (1 \oplus P) \oplus (P \oplus Q)) \oplus ((P \oplus Q) \oplus (Q \oplus 0)$

We know that XOR operator is associative

$\therefore$ $F(P, Q) = ((1 \oplus P) \oplus ((P \oplus Q) \oplus (P \oplus Q)) \oplus (Q \oplus 0)$

$= ((1 \oplus P) \oplus 0 \oplus (Q \oplus 0))$ $[\because (P \oplus Q) \oplus (P \oplus Q) = 0]$

$= (\overline{P} \oplus 0) \oplus (Q \oplus 0)$

$= \overline{P} \oplus Q$ $[\because A \oplus 0 = A,\ A \oplus 1 = \overline{A}\ ]$

$= \overline{P \oplus Q}$

$= P \odot Q$

$\therefore$ (D) is the answer.

GENRAL APTITUDE SECTION

1. The given sentence has an error of dangling modifier. The part IV i.e. 'was losing consciousness' does not go with the sentence. It should instead by 'lost his consciousness'

$\therefore$ (D) is the answer.

2. (C)

3. Coherent means logical and well organized or working closely and well together.

rambling means to move aimlessly from place to place.

$\therefore$ (C) is the answer.

4. The series is

2, 5, 10, 17, 26, 37, 50, 64

+3 +5 +7 +9 +11 +13 +15 = (64)

$\therefore$ 64 is the wrong term in the series. It should be 65 instead

(C) is the answer.

5.

Q.No.	Marks	Answered	Answered wrongly	Not attempted	Total
1	2	21	17	6	44
2	3	15	27	2	44
3	2	23	18	3	44

$\therefore$ No. of students = 44

$$\text{Average marks} = \frac{21\times 2+15\times 3+23\times 2}{44}$$

$$= 3.0227$$

$\therefore$ (C) is the answer.

6. (A) is inappropriate as only some students are participating and hence not all need to come at 9.00 a.m. Also the students who are participating should bring a parent along.

(B) is the appropriate instruction.

(C) is inappropriate as non participants have to come at 10:00 am. along with their parents.

(D) is inappropriate as participants have to come at 9:00 am and not before 9:00 am

$\therefore$ (B) is the answer.

7. Only (iv) can be concluded from the given passage.

$\therefore$ (D) is the answer.

8. Let the GDP in rupees be 'x'

Then its international value = $\frac{x}{50}$ (in dollars)

% increase = 7%

$\therefore$ GDP becomes $\frac{107x}{100}$

Then its international value = $\frac{107x}{100\times 60}$

$\therefore$ There is an overall decrease in the GDP

$$\text{\% decrease} = \frac{\left(\frac{x}{50}\right)-\left(\frac{107x}{6000}\right)}{\left(\frac{x}{50}\right)}\times 100$$

$$= 10.833\%$$

$$\approx 11\%$$

$\therefore$ (D) is the answer.

9. In 2011,

$$\frac{M}{F} = 1$$

In 2012,

$$\frac{M}{F} = \frac{3}{2}$$

Suppose the number of females in 2011 is x

$\therefore$ No. of males = x

Not it is given that no. of females in 2011 and 2012 is equal i.e.x

$\therefore$ Number of males = $\frac{3}{2} \times x$

$\therefore$ $\frac{\text{Males in 2012}}{\text{Males in 2011}} = \frac{3/2\,x}{x} = \frac{3}{2} = 1.5 : 1$

(C) is the answer.

10. $(7526)_8 - (Y)_8 = (4364)_8$

$\therefore$ $(Y)_8 = (7526)_8 - (4364)_8$

$= (3142)_8$

In subtraction using octal system, we borrow $(10)_8$ i.e. 8.

$\therefore$ (C) is the answer.